

# Maximum Gain Round Trips with Cost Constraints

Franz Graf, Hans-Peter Kriegel, Matthias Schubert

Institute for Informatics, Ludwig-Maximilians-Universität München, Oettingenstr. 67,  
D-80538 Munich, Germany  
{graf,kriegel,schubert,}@dbs.ifi.lmu.de

**Abstract.** Searching for optimal ways in a network is an important task in multiple application areas such as social networks, co-citation graphs or road networks. In the majority of applications, each edge in a network is associated with a certain cost and an optimal way minimizes the cost while fulfilling a certain property, e.g connecting a start and a destination node. In this paper, we want to extend pure cost networks to so-called cost-gain networks. In this type of network, each edge is additionally associated with a certain gain. Thus, a way having a certain cost additionally provides a certain gain. In the following, we will discuss the problem of finding ways providing maximal gain while costing less than a certain budget. An application for this type of problem is the round trip problem of a traveler: Given a certain amount of time, which is the best round trip traversing the most scenic landscape or visiting the most important sights? In the following, we distinguish two cases of the problem. The first does not control any redundant edges and the second allows a more sophisticated handling of edges occurring more than once. To answer the maximum round trip queries on a given graph data set, we propose unidirectional and bidirectional search algorithms. Both types of algorithms are tested for the use case named above on real world spatial networks.

## 1 Introduction

Searching for optimal ways in networks is an important task in many application areas. In most cases, an optimal way is a way minimizing the cost while fulfilling a certain property. The cost is usually connected to traversing the edges being part of the way. For example, in road networks the cost of each edge might be considered as the time it takes to traverse the edge. Finding the fastest route between two nodes can now be defined as finding the route with the property to connect both points and additionally having a minimum cost. In social network analysis, the cost is often measured in hops and the shortest path between two members in the network is considered as a measure for the strength of their relationship. Other application areas for networks might be co-citation networks and protein interaction networks where shortest paths can be used to express similarity. In all of these domains it is expected that traversing a certain edge is connected to a certain cost.

In this paper, we like to extend this view by additionally connecting the traversal of an edge to a certain amount of gain. In our running example, we consider a traveler who wants to take a hike in a nature resort. Usually, the traveler wants to see as much of the landscape as possible. Thus, walking on a scenic trail with a good view and passing by important landmarks represents some gain. However, our traveler can usually only walk for a certain period of time until (s)he wants to be back to the starting point, e.g. the parking lot. Thus, the task is to maximize the gain of our traveler while not spending more than a certain amount of time. Another application for the proposed query type is a car announcing a blood donation event at the local hospital. The hospital has hired the car with the speaker for a limited period of time only and now wants to find a route reaching as many people willing to donate blood as possible. Let us note that the problem can be easily extended to searching a way ending at a different destination than the starting point. Though we will mainly focus on the case of round trips, the proposed query processing can be easily extended to the more general case.

A round trip in our definition is a way and thus, it is allowed to pass by the nodes and edges more than once. Thus, it is different from finding Euler or Hamilton paths in a graph. An important issue about our problem definition is the possibility to traverse the same edge for an arbitrary amount of times. Depending on the given task, this might lead to round trips having a large redundancy in the number of visited edges. Therefore, we distinguish our problem further into two sub problems: The first allows edges to be visited several times and thus simply maximizes the gain while keeping the cost below the given threshold  $\tau$ . In the second setting, the definition of round trips is extended by allowing that each edge in the round trip is visited at most  $k$  times. Furthermore, each edge is considered only once when calculating the gain without consideration of any additional traversal.

To solve both problems, we will introduce algorithms that determine a maximum gain round trip for all cost budgets being smaller than a certain threshold  $\tau$ . To tackle this computationally complex task for practically relevant cost thresholds, we will introduce pruning mechanisms and bidirectional search methods. The proposed algorithms are evaluated by searching round trips on real world map data, obtained from OpenStreetMap.

The rest of the paper is organized as follows. Section 2 reviews related work. In section 3, we define preliminaries and our new queries. Section 4 and section 5 introduce pruning methods and search algorithms. Section 6 evaluates our algorithms on real world data w.r.t. runtime and various parameter settings. Finally, section 7 summarizes the paper and outlines directions for future research.

## 2 Related Work

Common route search which starts from a single source node to at least one target node is also known as the single-source problem. This problem has been studied very extensively for a long time [1,2,3,4,5,6,7,9,11,15,17]. Also the task

of finding not just the shortest or fastest route but the top  $k$  routes has achieved quite some interest and has been studied for several years e.g. in [16].

The closest scenario to the cost-gain networks discussed in this paper are multi-attribute or multi-cost networks that associate multiple types of cost to traversing a single edge, e.g. length and average speed. In [12], the authors introduce preference queries in such multi-cost networks. In particular, the paper proposes ranking and skyline queries to compute and sort a result set of possible target destinations in a multi-cost transport network. This work is different from the work presented in this paper because the query result consists of possible destination locations. Another related work is presented in [13]. In this work, the authors also work on a multi-cost or multi-attribute network and compute a skyline operator on the paths leading from a given starting point to a destination. The query result consists of all paths from the starting node to the destination node having a pareto optimal cost vector. The important difference to the queries in this paper consists in the use of gain attributes. Using gain has a major impact on the characteristics of the solutions. In cost and multi-cost networks optimal solutions always consist of shortest paths w.r.t. some possibly combined cost function. In a cost-gain network, the impact of traversing an edge needs to be measured w.r.t. the provided gain as well. Thus, optimal solutions do not need to be paths but can be ways. For example, the skyline operator proposed in [13] cannot generate interesting round trips because leaving an edge cannot decrease any cost value compared to the trivial solution of simply staying at the starting node. Thus, the trivial solution would always dominate any other way leading away from the starting node.

To the best of our knowledge, there exists no other work that formulates the search for optimal round trips while employing a cost-gain network. In theoretical computer science, there exist methods for finding all cycles present in a graph that will contain the most interesting round trips or parts of it. For example, [10] deals with the task of finding a complete cycle base in a graph. However, finding a complete cycle base is a different task and does not consider cost and gain values for the different edges.

### 3 Cost-Gain Networks, Round Trips and Queries

A network is represented by a graph where the edges have two attributes, cost and gain. Thus, we call this graph a cost-gain network:

**Definition 1 (Cost-Gain Network (CGN)).**

*A cost-gain network is a graph  $\mathcal{G}(V, E, cost, gain)$  where  $V$  is denoting the set of vertices and  $E \subset V \times V$  is denoting the set of edges.*

*$cost : E \rightarrow \mathbb{R}_+^0$  is called cost function where  $cost(e)$  denotes the non-negative cost for an edge  $e \in E$ .*

*$gain : E \rightarrow \mathbb{R}_+^0$  is called gain function where  $gain(e)$  denotes non-negative gain for an edge  $e \in E$ .*

In our running example, a CGN represents a network of roads, streets, paths, sidewalks and trails. The nodes correspond to crossings, the edges correspond to

path segments in the graph. The cost of a segment represents a certain combination of the characteristics of this segment, like the length, the maximum speed, the time needed to pass the segment, the inclination etc. The gain of a segment can be defined correspondingly by combining all characteristics that the user considers as beneficial, e.g. a trail that is not used by cars might be considered as more attractive for a hiker than a highway segment etc.

Let us also note that the attributes of an edge  $e_i = (n_s, n_d)$  need not be the same as edge  $\hat{e} = (n_d, n_s)$  even though both edges describe the exactly same path just in different directions as the user might for example avoid declines of a certain degree whilst accepting inclines of some other degree. Also the impact on travel speed for this edge might be different.

**Definition 2 (way).** *A way  $w$  is a sequence of edges  $((v_1, v_2), (v_2, v_3), \dots, (v_{k-1}, v_k))$  where the following condition holds:*

$$\forall 1 \leq i < k : \exists e \in E : e = (v_i, v_{i+1}) \quad (1)$$

*The cost of a way  $w$  is defined as follows:*

$$\text{cost}(w) = \sum_{i=1}^{k-1} \text{cost}((v_i, v_{i+1})) \quad (2)$$

*The gain of a way  $w$  is defined as follows:*

$$\text{gain}(w) = \sum_{i=1}^{k-1} \text{gain}((v_i, v_{i+1})) \quad (3)$$

In other words, a way is a sequence of connected edges. A round trip is a way starting and ending with the same node  $s$ .

**Definition 3 (round trip).** *A way  $w = ((v_1, v_2), (v_2, v_3), \dots, (v_{k-1}, v_k))$  is called round trip if  $v_1 = v_k$ .*

In the above definition of a round trip, there is no consideration of whether or not a round trip contains one and the same edge more than once. However, passing the same edge very often does not yield an additional benefit in most applications. For example, for a hiker looking for the most scenic walk, it might be necessary to pass the same edge twice, if some edges having a large gain are placed in a dead end. However, after visiting a spot and returning on the same path, it would not make any sense to go there again. Thus, a possibility to control the redundancy is to limit the number of times an edge can be contained in the round trip. Furthermore, traversing the same edge more than once should not contribute to the gain of the round trip. Thus, we can extend our definition of round trips to round trips with redundancy control. Formally, this can be formulated as follows:

**Definition 4 (Redundancy Control).** *Given the round trip  $r$  in the CGN  $G(V, E, \text{cost}, \text{gain})$ ,  $r$  is called a round trip under redundancy control with level  $k \in \mathbb{N}$ , if the following condition holds:*

$$\forall (v_a, v_b) \in r : |\{(v_i, v_{i+1}) \in r \mid (v_i = v_a \wedge v_{i+1} = v_b) \vee (v_i = v_b \wedge v_{i+1} = v_a)\}| \leq k$$

*For a round trip  $r$  under redundancy control, the gain is calculated as follows:*

$$\text{gain}(r) = \sum_{e \in ES(r)} \text{gain}((v_i, v_{i+1})) \quad \text{with} \quad ES(r) = \{e \in r\}$$

After defining both types of round trips, we can now define maximum gain round trip queries:

**Definition 5 (Maximum Gain Round Trip Query).**

*Given the CGN  $G(V, E, \text{cost}, \text{gain})$ , a starting node  $s \in V$  and cost threshold  $\tau \in \mathbb{R}^+$ , the result of a maximum gain round trip query (MGRQ) is the set  $R$  of round trips, such that for each element  $r \in R$  the following constraints hold:*

- (a)  $\text{cost}(r) \leq \tau$
- (b)  $\forall r, \hat{r} \in R : \text{gain}(r) < \text{gain}(\hat{r}) \Leftrightarrow \text{cost}(r) < \text{cost}(\hat{r})$

In other words, the result of a MGRQ contains a round trip providing maximum gain for each cost level being smaller than  $\tau$ . Thus, the result set contains all pareto optimal ways starting and ending at  $s$ .

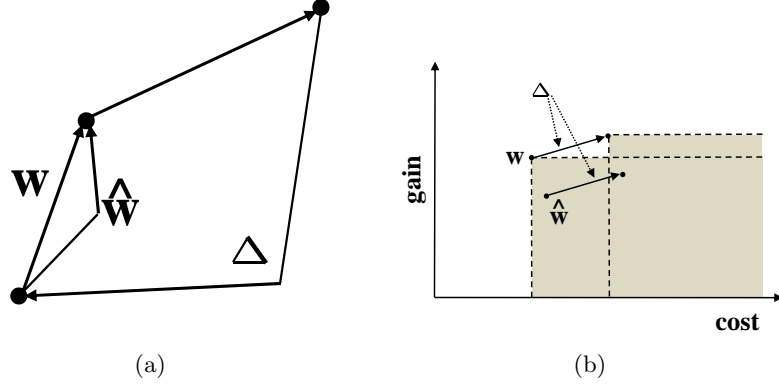
## 4 Algorithms for MGRQs

In this section, we will examine the problem of searching maximum gain round trips with cost constraints without controlling the redundancy.

### 4.1 Pruning Round Trips

The simplest pruning condition for a way  $w = ((v_{1,2}), \dots, (v_{k-1}, v_k))$  during our search is that the way cannot be extended into a round trip  $r = ((v_{1,2}), \dots, (v_{k-1}, v_k), \dots, (v_l, v_1))$  with  $\text{cost}(r) \leq \tau$ . Thus, if  $\text{cost}(w) > \tau$ ,  $w$  can be pruned because the cost of any extension of  $w$  must be larger than  $\text{cost}(w)$ .

Let us note that pruning w.r.t. gain is not that simple. Extending a way usually increases the gain and thus, there is no upper limit for the gain of a way. However, since we are only looking for round trips having an optimal cost-gain ratio, we can use the following observation for defining a further pruning criterion: For each  $r \in R$ , we can guarantee that there is no other round trip generating more gain and having at most the same cost. Therefore, each partition  $w$  of the result round trip  $r$  has a pareto optimal cost gain ratio w.r.t all other ways starting and ending at the same position as  $w$ . The intuition behind this conclusion is that each part of the round trip serves three purposes: it moves the traveler to the end of the way, it generates a certain amount of cost and it provides a certain amount of gain. Apart from this, the properties of the way do not influence the properties of the complete round trip. Formally, we can define the following pruning rule based on the domination relationship:



**Fig. 1.** Local pruning of pareto optimal paths: if  $\hat{w}$  is dominated by  $w$ , any further extension  $\Delta$  of  $w$  and  $\hat{w}$  does not change the pruning relation so that  $\hat{w} + \Delta$  is still dominated by  $w + \Delta$ .

**Lemma 1 (local pruning by domination).** *Let  $w = ((v_1, v_2), \dots, (v_{k-1}, v_k))$  be a way in  $G(E, V, \text{cost}, \text{gain})$ , then  $w$  is dominated by another way  $\hat{w} = ((v_1, \hat{v}_2), \dots, (v_{k-1}, v_k))$  if either  $\text{cost}(w) > \text{cost}(\hat{w}) \wedge \text{gain}(w) \leq \text{gain}(\hat{w})$  or  $\text{cost}(w) \geq \text{cost}(\hat{w}) \wedge \text{gain}(w) < \text{gain}(\hat{w})$ . If  $w$  is dominated by  $\hat{w}$ , then  $w$  cannot be extended into an element of the result set of an MGRQ for  $v_1$ .*

*Proof.* Consider the result round trip  $r$  with  $\text{gain}(r)$  and  $\text{cost}(r)$ . Due to the definition of the result set, we can rule out that there is another round trip  $\hat{r}$  having at most the same cost and more gain or at least the same gain and less cost. Now, consider a way  $w = (v_1, v_2, \dots, v_i)$  being part of  $r$ . If there exists another way  $\hat{w} = (v_1, \hat{v}_2, \dots, v_i)$  in  $r$  leading from  $v_1$  to  $v_i$  with  $\text{cost}(\hat{w}) < \text{cost}(w) \wedge \text{gain}(\hat{w}) \geq \text{gain}(w)$  or  $\text{cost}(\hat{w}) \leq \text{cost}(w) \wedge \text{gain}(\hat{w}) > \text{gain}(w)$ , then it is possible to construct a round trip  $r_{\text{new}}$  by replacing  $w$  by  $\hat{w}$  in  $r$ . However, in this case  $\hat{r}$  would dominate  $r$  which contradicts the condition that  $r$  is part of the result set.

An illustration of this pruning mechanism can be found in figure 1.

## 4.2 A Basic MGEQ Algorithm

In our descriptions, we will denote the edges starting at node  $v$  as outlinks of  $v$  while the edges ending at  $v$  are called inlinks. Our algorithm employs two data structures. The first is a hash table called *node tab* containing an entry for each visited node  $v_i$ . Furthermore, the node tab stores all undominated ways starting at  $s$  and ending at  $v_i$ . For each of these ways, we store a flag indicating whether we already processed the way in a previous step or not. In the following, we will use the expression "update the node tab with way  $w$ " for the following steps being used in our algorithm:

1. Check whether  $w$  is dominated by any entry of the node  $tab$ .
2. If  $w$  is not dominated, insert  $w$  into the node  $tab$  entry.
3. Remove all entries from the node  $tab$  which are dominated by  $w$ .

Our second data structure is a priority queue containing all nodes. Each node  $v_i$  is prioritized by the maximum gain among all ways ending at  $v_i$  and the queue is organized in descending order.

```

SimpleMGRQ(Node  $s$ , Float  $\tau$ )
(1) NodeTab  $tab$  = InitNodeTab()
(2) PriorityQueue  $queue$  = InitQueue()
(3) FOR EACH Link  $l$  IN  $s.outlinks()$  DO
(4)   Way  $w$  = new Way( $s, l$ ) (5)    $tab.update(w)$ 
(6)    $queue.update(w.last, w.gain)$ 
(7) END FOR
(8) WHILE NOT  $queue.isEmpty()$  DO
(9)   Entry  $entry$  =  $queue.pop()$ 
(10)  List<Way>  $aktList$  =  $entry.getUndominated()$ 
(11)   $aktList$  =  $removeProcessedWays(aktList)$ 
(12)   $setProcessed(aktList)$ 
(13)  List<Way>  $candidates$  =  $extendWays(aktList)$ 
(14)  FOR EACH  $w$  IN  $candidates$  DO
(15)    IF  $w.cost < \tau$  DO
(16)       $tab.update(w)$ 
(17)       $queue.update(w.last, w.gain)$ 
(18)    END IF
(19)  END FOR
(20) END WHILE
(21) RETURN  $tab.getEntry(s)$ 

```

**Fig. 2.** Pseudocode of the simple MGRQ Algorithm

The algorithm starts by generating the ways resulting from following all out links of the starting node  $s$ . Afterwards the ways are used to update the node  $tab$  as well as the queue. Now the algorithm enters the main loop which is repeated until the priority queue is empty. In each iteration, the algorithm pops the top node from the queue and retrieves all unprocessed ways from the node  $tab$ . Let us note that we have to keep already processed ways in the node  $tab$  for determining locally dominated ways. However, it is not required to process each way more than once. Now, each unprocessed way is marked as processed. Afterwards we extend each way by all of its out links generating a set of candidate ways. The candidate ways are checked whether their cost exceeds the limit  $\tau$ . Afterwards each candidate  $c = ((s, v_1), \dots, (v_k, v_{new}))$  is checked against the ways being stored in the node  $tab$  entry of their end node  $v_{new}$ . If  $c$  is not dominated by any other way, it is inserted into the node  $tab$  entry. Furthermore, if  $c$  dominates former members of the node  $tab$  entry, these members can be pruned due to their sub optimal cost gain ratio. If the maximum gain of any node  $tab$  entry being modified is increased, the entry has to be updated in the queue. After the queue is empty, the result of our query can be found in the node  $tab$  entry of the starting node  $s$ . Figure 2 displays the algorithm in pseudo code.

Let us note that the above algorithm is capable to find arbitrary pareto optimal ways having a cost less than  $\tau$  and ending at any visited node. Thus, it is not restricted to the search of round trips.

```

BidirectionalMGRQ(Node s, Float  $\tau$ )
(1) NodeTab tab = InitNodeTab();
(2) PriorityQueue queue = InitQueue()
(3) FOR EACH Link l IN s.outlinks() DO
(4)   Way w = new Way(s, l)
(5)   tab.updateStart(w)
(6)   queue.update(w.last, w.gain)
(7) END FOR
(8) FOR EACH Link l IN s.inlinks() DO
(9)   Way w = new Way(s, l)
(10)  w = w.reverse()
(11)  tab.updateReturn(w)
(12)  queue.update(w.first, w.gain)
(13) END FOR
(14) WHILE NOT queue.isEmpty() DO
(15)   Entry entry = queue.pop()
(16)   List<Way> fwdList = entry.getundominatedStart()
(17)   fwdList = removeProcessed(fwdList)
(18)   setProcessed(fwdList)
(19)   List<Way> bwdList = entry.getundominatedReturn()
(20)   bwdList = removeProcessed(bwdList)
(21)   setProcessed(bwdList)
(22)   FOR EACH w IN fwdList DO
(23)     IF w.cost >  $\frac{\tau}{2}$  DO
(24)       fwdList.delete(w)
(25)     END IF
(26)   END FOR
(27)   List< Ways > fwdCandidates = extendFwdWays(fwdList)
(28)   FOR EACH w IN fwdCandidates DO
(29)     tab.updateStart(w)
(30)     queue.update(w.last(), w.gain)
(31)   END FOR
(32)   List<Ways> bwdCandidates = extendbwdWays(bwdList)
(33)   FOR EACH w IN bwdCandidates DO
(34)     IF w.cost <  $\frac{\tau}{2}$  DO
(35)       tab.updateReturn(w)
(36)       queue.update(w.last, w.gain)
(37)     END IF
(38)   END FOR
(39) END WHILE
(40) LIST< WAYS > result = new Listjwaysi()
(41) FOR EACH entry IN tab DO
(42)   FOR EACH startWay IN entry.getundominatedStart() DO
(43)     FOR EACH retWay IN entry.getundominatedReturn() DO
(44)       Way roundtrip = startWay.extend(retWay)
(45)       result.update(roundtrip)
(46)     END DO
(47)   END DO
(48) END FOR
(49) RETURN result.entries

```

**Fig. 3.** Pseudocode of the bidirectional MGRQ Algorithm



### 4.3 Bidirectional Round Trip search

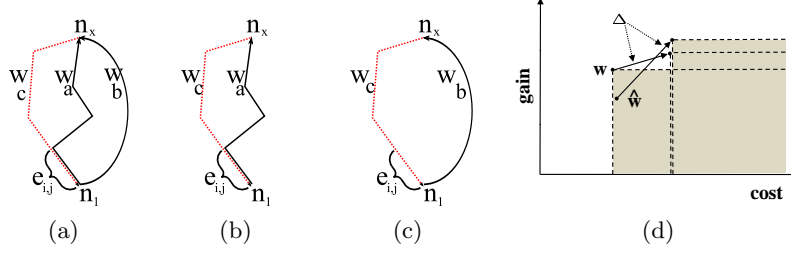
A further method to improve the runtime is bidirectional search as employed in the well-known bidirectional Dijkstra search for shortest paths [14]. For searching maximum gain round trips, bidirectional search yields an even stronger advantage. The algorithm described above generates ways having a cost of at most  $\tau$ . Thus, it has to visit any node that is reachable by spending the cost limit  $\tau$ . However, a round trip has to end at its starting node. Obviously, examining a way  $w$  having a network distance of  $\tau$  cannot have a return path  $w_2$  to the starting node that would not exceed  $\tau$ . Thus, it is only necessary to explore ways  $w_1$  for which there exists a return path  $w_2$  having at most a cost of  $cost(w_2) = \tau - cost(w_1)$ .

To conclude, it is only necessary to extend each way  $w$  until  $cost(w)$  exceeds  $\frac{\tau}{2}$ . In particular, we can distinguish two cases, when trying to split  $w$  into two partitions of equal cost. In the first case, there is a node after the distance of exactly  $\frac{\tau}{2}$ . In the second case, the split point having exactly the cost of  $\frac{\tau}{2}$  is located at the edge  $e = (v_i, v_{i+1})$ . Then, there is a unique partitioning of  $w$  into three parts  $w_1, e$  and  $w_2$  and by extending  $w_1$  by  $e$ , we will get a unique partitioning of  $w$  into starting way  $w_1$  and return way  $w_2$ . Based on this observation, we can stop extending ways that exceed the cost limit of  $\frac{\tau}{2}$  at most by one hop and thus, approximately work with only half of the search radius.

Though bidirectional search is applicable for searching any maximum gain way  $w$  having  $cost(w) < \tau$ , it is especially well suited for searching round trips. Since the area of the graph that has to be explored for finding the starting ways is the same as the area being explored for finding the return ways, it is possible to simultaneously search for both parts of a round trip. Thus, the part of the graph being accessed during query processing is significantly decreased.

Our bidirectional search algorithm employs a node tab storing pareto optimal ways leading to a visited node  $v$  managing two lists of undominated ways. The first contains all undominated ways  $w_i$  leading from the starting node  $s$  to  $v$  and the second manages all undominated ways starting at  $v$  and leading to  $s$ . Updating the node tab is used in an almost identical way as described above. The only difference to the above use is that we have to distinguish whether  $w$  is a starting way or a return way and update the corresponding list. The processing order is again managed by a priority queue that is ordered by the maximum gain being observed in either part of the node tab.

The algorithm proceeds as follows: At initialization, the algorithm considers all outlinks of the starting node  $s$  and generates a first set of starting ways. After updating the node tab with these ways and inserting the corresponding nodes into the priority queue, we use all inlinks to  $s$  to generate a first set of return ways and again update the node tab and the priority queue. Now the algorithm enters the main loop which is iterated until the priority queue is empty. In each iteration  $i$ , the algorithm pops the top node  $v_i$  from the queue. Afterwards, it retrieves all unprocessed ways from the list of starting ways leading to  $v_i$  in the node tab and checks if these have a cost smaller than  $\frac{\tau}{2}$ . If a way  $w$  is still smaller than  $\frac{\tau}{2}$ , the algorithm extends  $w$  by all outlinks of  $v_i$  and generates a candidate set  $C_w$ . Each element of  $c \in C_w$  is now used to update the node tab and the



**Fig. 4.** Figure that indicates the problem of local pruning if an edge cardinality is constrained.  $\Delta$  indicates the cost/gain of  $p_c$  which is added to the paths  $p_a, p_b$ . The cost contribution to  $p_a$  and  $p_b$  is the same with different gain because  $p_c$  and  $p_a$  share an edge. And thus, its pruning power is lowered as well.

priority queue in case the maximum gain of the node tab entry of the last node  $c$  is increased. Afterwards the algorithm retrieves all unprocessed return ways. Each of these ways  $w_{ret}$  is extended by all inlinks to a set of candidate ways  $C_{w_{ret}}$  of ways starting at  $v_i$  and ending at the origin  $s$ . Then each candidate  $c \in C_{w_{ret}}$  is checked whether its cost is still less or equal  $\frac{\tau}{2}$ . If  $c$  passes this test,  $c$  is used to update the node tab and the queue, if the maximum gain of the entry of its first node is increased. Let us note that it is important to check starting ways and return ways at different stages of the processing to achieve all ways sufficing the partitioning described above. After the priority queue is empty, i.e. there is no unprocessed way left that can be extended any further, we need to join the pareto optimal starting ways and return ways. The result set is organized in a list of undominated round trips which is updated by visiting all entries of the node tab. For each entry representing node  $v$ , we examine all combinations of undominated ways starting at  $w_{start}$  and return ways  $w_{ret}$ . For each pair of ways  $w_{start}$  and  $w_{ret}$ , we first of all determine the cost of the corresponding round trip by adding  $cost(w_{start}) + cost(w_{ret})$  and the corresponding gain by adding  $gain(w_{start}) + gain(w_{ret})$ . Based on this cost-gain vector, we can now update the result list of undominated round trips. If the new round trip is undominated, we join both ways and add the result to the result list. If the new round trip even dominates formerly pareto optimal round trips, the now dominated round trips are deleted. After each entry of the node tab is processed, the algorithm terminates and the result consists of all pareto optimal round trips with a cost of up to  $\tau$ . Figure 3 describes the bidirectional search in pseudo code.

## 5 MGRQs with Redundancy Control

In this section, we will discuss maximum gain round trip queries under the constraints limiting the amount and the impact of edges which occur more than once. In particular, we will not allow that a round trip contains the same edge more than  $k$  times. Furthermore, as the cost will sum up over duplicate edges

as well, the gain of ways is calculated over the set of all edges being contained in the round trip. Since there are no duplicates in a set, each edge can add its gain only once to the round trip. Please recall that we will consider edge  $(v_i, v_j)$  as equivalent to  $(v_j, v_i)$  w.r.t. redundancy control.

### 5.1 Pruning and Redundancy Control

Since computing the cost is the same in both types of round trips, pruning ways w.r.t. their cost is applicable in the same way as described in Section 5. However, when trying to exploit the cost-gain ratio to prune ways, both redundancy control mechanisms have a major impact. When using redundancy control, we cannot guarantee that all partitions of a pareto optimal round trip  $r$  are pareto optimal in the same ways as described above. Figure 4 illustrates this effect in a simple example. Even though the way  $w$  dominates the way  $\hat{w}$ ,  $w$  is not part of the pareto optimal round trip  $r$ . However, the dominated way  $\hat{w}$  is part of  $r$  and replacing  $\hat{w}$  by  $w$  would lead to a round trip  $\hat{r}$  being dominated by  $r$ . In this example we can observe that the influence of  $w$  to the gain of the complete round trip  $r$  is not limited to the  $cost(w)$ ,  $gain(w)$  and its end node  $v$ . If  $w$  is extended into a round trip, all edges being visited on  $w$  will not add any gain and thus, these edges influence the cost gain ratio of the complete round trip. A similar observation holds for limiting the cardinality of each edge in a round trip to  $k$ . In this case, a pareto optimal round trip  $r$  might contain the dominated way  $w$  because the way  $\hat{w}$  dominating  $w$  contains edges that would occur more than  $k$  times when replacing  $w$  by  $\hat{w}$  in  $r$ . In other words, the corresponding round trip  $\hat{r}$  containing  $\hat{w}$  would violate the redundancy control.

In order to retain the possibility of local pruning, the domination relation defined above must be extended by the set of visited edges. Thus, to extend local pruning to redundancy controlled round trips, we can formulate the following lemma:

**Lemma 2 (domination under redundancy control).**

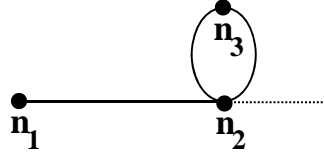
Let  $\hat{w} = ((v_1, \hat{v}_2), \dots, (\hat{v}_{k-1}, v_k))$  be a way in  $G(E, V, cost, gain)$ , then  $\hat{w}$  is called dominated under redundancy control by another node  $w = ((v_1, v_2), \dots, (v_{k-1}, v_k))$  if the following conditions hold:

- (a)  $(cost(\hat{w}) > cost(w) \wedge gain(\hat{w}) \leq gain(w))$   
 $\vee (cost(\hat{w}) \geq cost(w) \wedge gain(\hat{w}) < gain(w))$
- (b) For  $ES(w) = \{e \in E | e \in w\}, ES(\hat{w}) = \{e \in E | e \in \hat{w}\}: ES(w) \subseteq ES(\hat{w})$

If  $\hat{w}$  is dominated under redundancy control by  $w$ , then  $\hat{w}$  cannot be extended into an element of the result set of an MGRQ for  $v_1$ .

*Proof.* The proof for condition (a) is identical to the proof in section 4. It remains to show that replacing  $\hat{w}$  in a round trip  $\hat{r}$  by  $w$  cannot lead to a reduced gain due to duplicate edges or an invalid round trip. Consider the way  $w_{ret}$  extending  $\hat{w}$  to the way  $\hat{r}$ . The additional gain being earned by traversing  $w_{ret}$  in  $\hat{r}$  can be described as

$$gain(w_{ret}) - \sum_{e \in ES(\hat{w}) \cap ES(w_{ret})} gain(e)$$



**Fig. 5.** This figure depicts a way  $(n_1, n_2, n_3, n_2)$  that can be pruned if  $\text{gain}((n_2, n_3, n_2)) = 0$ . In any case, additional traversals of the cycle  $(n_2, n_3, n_2)$  do not yield any gain. Thus, the way  $(n_1, n_2, n_3, n_2, n_3, n_2)$  would be pruned in any case.

Since  $\text{gain}(e) \geq 0$  and  $ES(w) \cap ES(w_{ret}) \subseteq ES(\hat{w}) \cap ES(w_{ret})$ ,  $\text{gain}(r) \geq \text{gain}(\hat{r})$ . Furthermore, if  $\hat{r}$  does not violate the redundancy parameter  $k$ , then  $r$  cannot violate it either because it follows from  $ES(w) \subseteq ES(\hat{w})$  that  $w$  does not contain any edge  $e$  with  $e \notin \hat{w}$ . Thus, any violation in  $r$  would be encountered in  $\hat{r}$  as well.

A major issue for the usefulness of this lemma is whether there are enough ways that can be pruned to justify the additional effort for comparing the edge sets. In the following, we start off by discussing the worst case scenario and afterwards point out the cases in which our pruning rule still justifies the overhead.

For values of  $k \leq 2$  and  $\text{gain}(e) > 0$  for each  $e \in E$ , it is impossible that any way can be pruned. Since the dominated way  $\hat{w}$  must contain at least the same edges as  $w$ ,  $\text{gain}(\hat{w}) \geq \text{gain}(w)$ . Due to  $w$  dominating  $\hat{w}$ , we know  $\text{gain}(w) = \text{gain}(\hat{w})$  and  $\text{cost}(w) < \text{cost}(\hat{w})$ . Based on this observation, it follows that  $w$  and  $\hat{w}$  must have the same set of edges, i.e.  $ES(w) = ES(\hat{w})$  because  $\forall e \in E : \text{gain}(e) > 0$  and  $\text{gain}(w) = \text{gain}(\hat{w})$ . Thus,  $\hat{w}$  cannot contain an additional edge to  $w$ . Thus, the only allowed difference between  $\hat{w}$  and  $w$  is that  $\hat{w}$  visits some edges  $e \in ES(w)$  more than once. For  $k = 2$  the only possibility that  $w$  and  $\hat{w}$  end with the same node is  $w = \hat{w}$ .

Correspondingly, for  $k > 2$  and  $\forall e \in E : \text{gain}(e) \geq 0$ , there might be ways  $\hat{w}$  being dominated by  $w$ . In figure 5, we illustrate the types of ways being pruned. On the left side  $\hat{w}$  extends  $w$  by a cycle consisting of new edges offering no gain. The example on the right hand side excludes the way  $\hat{w}$  because it visits a cycle in  $w$  more than once. To subsume, domination under redundancy control prunes all ways containing cycles which do not provide gain.

A final pruning mechanism we have to consider is the redundancy parameter  $k$ . Since we can prune all ways violating the cardinality threshold  $k$ , the number of ways we have to consider for further extension is usually much smaller than in the general case without redundancy control. The smaller the value of  $k$  is chosen the stronger is its pruning power.

To conclude, searching for maximum gain round trips under redundancy control can employ cost-based pruning as in Section 4. Additionally, choosing a small parameter value  $k$  also has the power to prune invalid ways during the traversal. However, when increasing the value of  $k$ , the number of pruned paths is

strongly decreasing. For parameter settings, domination-based pruning justifies the overhead because it prevents the algorithms from exploring ways which are revisiting identical parts of the way multiple times.

## 5.2 Algorithm for MGRQs with Redundancy Control

Both algorithms for answering MGRQs described in section 4 are easily adaptable for the case of redundancy controlled round trips. To modify the algorithms, we first of all have to integrate the redundancy control. Therefore, the gain calculation has to be changed in order to prevent duplicate edges to add any further gain. Furthermore, each time the algorithm tries to extend a way  $w$  by an edge  $(v_i, v_j)$ , we have to check whether  $(v_i, v_j)$  or its complement  $(v_j, v_i)$  is already contained in  $w$  more than  $k$  times. This way, all invalid candidates are already pruned before they are constructed in the first place.

In our case, we want to use domination under redundancy control as described above. The method for checking domination must be extended by additionally checking for the subset condition. However, if we employ small values of  $k$  and the majority of edges has a non-zero gain, it often makes sense to abandon pruning based on domination to avoid the computational overhead. In this case, the list of ways in the node tab entry for node  $v$  contains all valid ways found so far.

A final difference for the first algorithm described in Section 4 is that the result set has to be stored in a dedicated list of pareto optimal ways. In the previous setting, the node tab entry of the starting node  $s$  already contains a pareto optimal list of round trips. However, since the pruning rules under redundancy control are less restrictive, the remaining dominated round trips must be removed before returning the result set. Let us note that this difference has no impact to the bidirectional algorithm because the last step joining starting ways and return ways has to construct a new pareto optimal result set anyway. After implementing the named modifications, we can employ both algorithms to compute MGRQs under redundancy control.

## 6 Experimental Evaluation

In our evaluation, we use data obtained from OpenStreetMap<sup>1</sup>. We preprocessed the data using the converter provided in [8] to remove some of the nodes having a degree of 2. In our tests, we examined three different areas which are popular for hiking: Kirchsee (GER), Jasper(AL,CA) and Grand Canyon Village(AZ,US). For each area, we selected a central starting point<sup>2</sup>.

<sup>1</sup> Data and Map data © OpenStreetMap (and) contributors, CC-BY-SA  
<http://www.OpenStreetMap.org>

<sup>2</sup> Kirchsee: <http://www.openstreetmap.org/?node=312519650>  
 Jasper: <http://www.openstreetmap.org/?node=915165849>  
 Grand Canyon: <http://www.openstreetmap.org/?node=174618876>

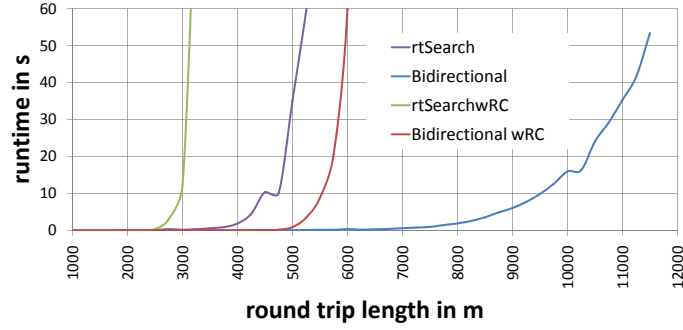
For all our tests, we chose the Euclidean distance between two nodes as a cost criterion. To represent the gain, we considered the road type. Thus, we assigned each edge allowing a maximum speed of less than 30 km/h with the gain of 1 and for the rest of the edges, we assigned a gain of 0.

*Processing time.* In our first set of experiments, we compare the runtime of the round trip search with redundancy control (*rtSearchwRC*) to the search allowing redundancy (*rtSearch*). Furthermore, we compare the bidirectional search (*bidirectionalRTS*) and the bidirectional search with redundancy control (*bidirectionalRTSwRC*). For *rtSearchwRC* and *bidirectionalRTSwRC* the redundancy parameter  $k$  was set to 1.

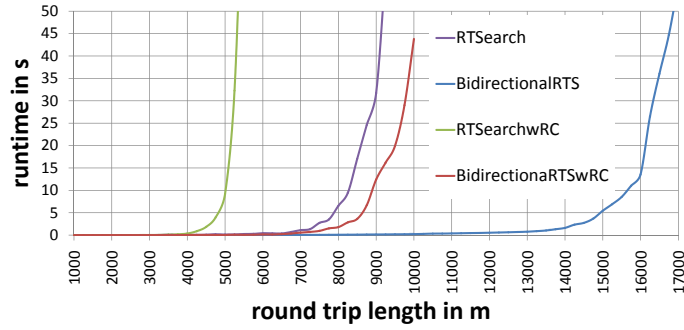
Figure 6 displays the runtime of all four algorithms with increasing cost values for all three maps. A first observation is that all algorithms display a certain cost budget for which the search starts to show a super linear increase in search time. However, we can observe that for employing bidirectional search the threshold for which query processing takes less than a minute can be extended to a reasonably large distance for round trips, large enough to be interesting (up to 17 km). Furthermore, on all graphs the bidirectional search could extend the cost limit being processable in less than one minute in comparison to the unidirectional algorithm for the same query type. A final conclusion that can be drawn from the results is that queries without redundancy control can be processed much faster than those employing redundancy control. This observation can be explained by the fact that dominance on the cost-gain graph is a much stronger pruning mechanism than dominance under redundancy control. Later on, we will present an experiment showing that dominance under redundancy control is very important for larger values of  $k$ .

*Search space.* Another important factor when searching in graph data is the portion of the graph which has to be available in main memory. Thus we examined the increase of the nodes visited during the search with *bidirectionalRTS* and *bidirectionalRTSwRC* w.r.t. the cost threshold  $\tau$ . The results for all three maps and both bidirectional search algorithms is displayed in figure 7. It can be seen that the portion of the graph visited increases approximately linearly with the threshold parameter  $\tau$ . Furthermore, we can observe that *bidirectionalRTS* and *bidirectionalRTSwRC* visit comparable portion of the graph. Let us note that 7(b) is scaled differently. Since the maximum cost threshold processable for *bidirectionalRTSwRC* is smaller than for *bidirectionalRTS*, we could not generate values for all distance thresholds we displayed in figure 7(a). To conclude, MGRTQs visit rather small portions of the graph. The high complexity of MGRTQs is rather caused by the large amount of possible round trips than by the size of the graph.

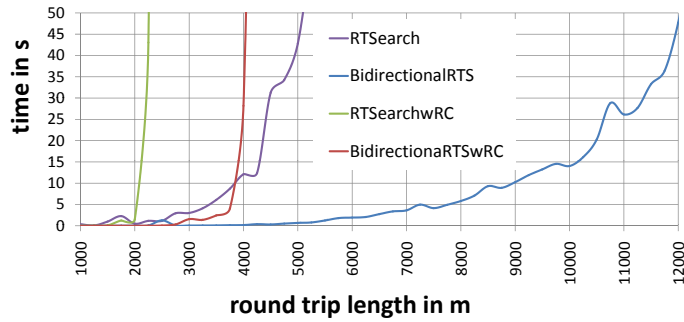
*Impact of the Redundancy Level  $k$ .* Another impact factor on the search speed when using redundancy control is  $k$ , which limits the cardinality of an edge. Figure 8 illustrates the impact of  $k$  on the retrieval times of *bidirectionalRTSwRC* on the Jasper map with varying  $k$  from 1 to 6. At first it can be observed that



(a) Kirchsee



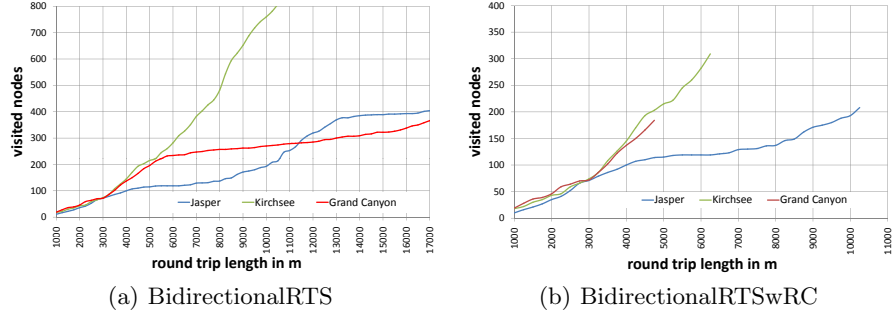
(b) Jasper (AL,CA)



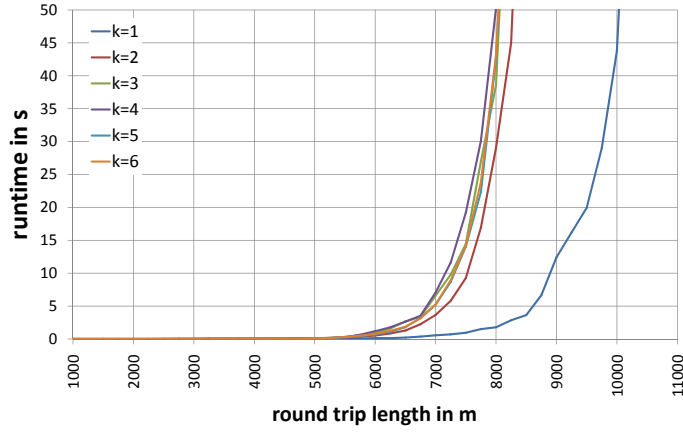
(c) Grand Canyon Village (AZ,US)

**Fig. 6.** Figures 6(a), 6(c) and 6(b) show the runtime of the proposed algorithms. For *rtSearchwRC* and *bidirectionalRTSwRC*  $k$  was set to 1.

setting  $k=1$  displays better runtimes and thus, the distance threshold which can still be computed ended at 10 km. For values of  $k \geq 2$ , the maximum threshold which could be reached was around 8 km. An interesting result is that for all  $k \geq 2$  the run time was approximately the same. Thus, with the exception of the



**Fig. 7.** Number of nodes visited by BidirectionalRTS (7(a)) and BidirectionalRTSwRC (7(b)) for all three maps.

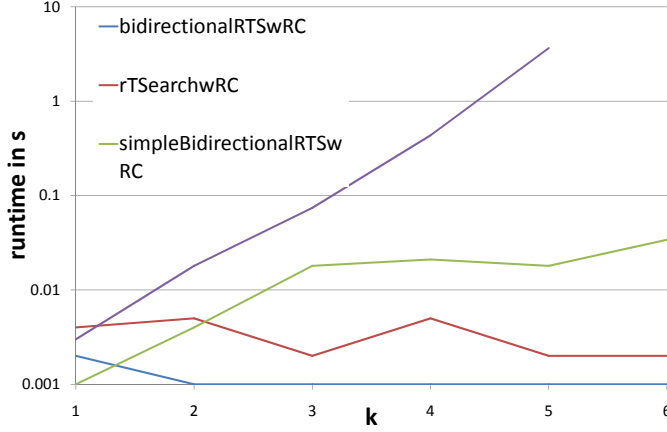


**Fig. 8.** Runtime of *bidirectionalRTSwRC* for different values of  $k$

special case  $k = 1$  the value of  $k$  does not have a strong influence on the runtime. The reason for this effect can be found in the pruning rule employing dominance under redundancy control. We will demonstrate this effect more clearly in the next experiment.

*Domination under Redundancy Control.* In Section 5, we proposed domination under redundancy control to implement an additional pruning rule. We already explained why the impact of this rule strongly depends on the value of the redundancy parameter  $k$ . In the following, we will examine the runtime behaviour of our search algorithms with dominance pruning (*rtSearchwRC*) and (*bidirectionalRTSwRC*) and search algorithms without the dominance pruning rule *simpleRTSwRC* and *simpleBidirectionalRTSwRC* for increasing values of  $k$ . The result is displayed in figure 9. The experiment was executed on the Jasper map with  $\tau = 1700\text{ m}$  and  $k$  varying from 1 to 6. Let us note that the relatively small





**Fig. 9.** Impact of redundancy control on runtime when executed on the Jasper map with  $\tau = 1700m$  and  $k$  varying from 1 to 6.

value for  $\tau$  was chosen to still be able to compute results with *simpleRTSwRC* and *simpleBidirectionalRTSwRC* for larger values of  $k$ .

In case of  $k = 1$ , our pruning rule cannot have any impact. Thus, it can be seen that the algorithms without the additional pruning rule perform better for values of  $k = 1$ . For increasing values of  $k$ , we can observe an exponential increase of the runtime for the basic algorithms *simpleRTSwRC* and *simpleBidirectionalRTSwRC*. In contrast, *rtSearchwRC* and *bidirectionalRTSwRC* show an almost constant runtime behaviour for increasing values of  $k$ . Thus, employing dominance pruning under redundancy control is a significant improvement for larger values of  $k$ .

## 7 Conclusions

In this paper, we examined maximum gain round trip queries (MGRQs) with cost constraints in cost-gain networks. A cost-gain network is a graph where each edge is connected to a certain amount of cost and additionally provides a certain amount of gain. A round trip is a way starting and ending at the same node and a maximum gain round trip is a round trip providing the maximum gain for a certain cost. The result set of an MGRQ is a set of round trips containing a maximum gain round trip for every cost level being less or equal to a cost limit  $\tau$ . We propose solutions for two sub problems. The first deals with round trips where edges might occur multiple times. The second sub problem restricts the number of times an edge can occur in the solution to a maximum value of  $k$ . Our algorithms are tested on real world map data taken from Open Street Map. For future work, we plan to examine further pruning mechanisms which are based on optimistic forward approximations similar to A\*-search. Furthermore, we plan

to examine parallel algorithms to extend the cost limit being still computable. Furthermore, we will develop approximative algorithms for the case that an exact search requires too much resources.

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